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On Errors in the Virtual Population Analysis.

by

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1. Abstract.

Errors introduced in the Virtual Population Analysis (VPA) due to uncertainties in the natural mortality are investigated. A bias on the estimates of the fishing mortality coefficient F, from the VPA of 25% may be expected.

Sampling error in catch will introduce error in the estimate of the mean fishing mortality coefficient $\bar{F}$. The relative error in F is about half the relative error of that found in the catches.

A method for estimating M and $\bar{F}$ using least square fit is given.

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Contents:

2. Definition of symbols used:

M is the instantaneous coefficient of natural mortality.

F is the instantaneous coefficient of fishing mortality.

Z is the instantaneous coefficient of total mortality.

N_i is the total number of fish of a year class at the i'th birthday.

C_i^y is the catch in number during year y of a year class i.

V_i is the virtual population in the year i, i.e. the total number of fish of a yearclass, which will be caught in the year i and subsequently.

E is the exploitation rate.

t-1 is the last age group where catches are separated.

$\mathcal{F}$ true value of F.

$\mathcal{M}$ true value of M.

$\mathcal{N}$ true value of N.

$\varrho(N_i)$, $\varrho(F_i)$ relative error in N_i and F_i respectively.

$\bar{F}_a^y$ Mean of fishing mortality coefficients for fish a years and older at year y.

3 Introduction

The Virtual Population Analysis (VPA)(Gulland 1965) is a method of estimating the stock size and the fishing mortality of a total exploited year class as a function of time (or age).

Below a brief rewiev of the method is given.

Let $$N_i = \frac{V_i}{E_i}$$

and
$$N_{i+1} = N_i e^{-Z_i} = \frac{V_{i+1}}{E_{i+1}} . \qquad (1)$$

The catch in number during year i is given by:

$$C_i = N_i E_i (1 - e^{-Z_i}). \qquad (2)$$

Taking the ratio (1)/(2) we get

$$\frac{V_{i+1}}{E_{i+1} C_i} = \frac{e^{-Z_i}}{E_i (1 - e^{-Z_i})} \qquad (3)$$

with $Z_i = F_i + M$.

F_i and N_i can be estimated from (3) when C_i, V_{i+1} and E_{i+1} are known. However, E_t, the exploitation rate for the oldest age-group of a year class, must be assumed, as data are not available.

The exploitation rate E_i for fish alive at the beginning of year i, may be calculated as the sum of proportions of fish caught during that year and those caught later.

$$E_i = E_i (1 - e^{-Z_i}) + E_{i+1} e^{-Z_i} .$$

Pope (1971) has developed a modification (the cohort analysis) of VPA, based on the approximation

$$\frac{\sinh F/2}{\sinh (F+M)/2} \approx \frac{F}{F + M}$$

which, according to Pope, is usable up to values of M =.3 and F=1.2 . He derived simple expressions for calculating the fishing mortality coefficient F_i and stock size N_i:

$$F_i = \ln(N_i / N_{i+1}) - M$$

and

$$N_i = C_i e^{M/2} + N_{i+1} e^{M} .$$

The advantage of using the VPA is, that F in a fishery, where F_i is changing in time may be estimated for a given age-group in a given year without knowing effort data.

The main disadvantage is, that unknown and often considerable errors may be introduced due to uncertainties of M and F_t.

Pope (1971) has discussed errors in F_i and N_i arising from incorrect choice of F_t (or equivalent E_t) and sampling errors of C_i.

In this paper errors caused by uncertainty in M will be considered and an attempt to reduce this error is described (4.3). An extension of the VPA by computing the average fishing mortality coefficient for the fully recruited age-groups is discussed (4.2).

In his paper Pope has shown

$$\rho(N_i) = \rho(N_{i+1})e^{-\mathcal{F}_i} \qquad (4)$$

and the $\rho(F_i)$ can be evaluated as follows

$$F_i - \mathcal{F}_i = \ln(N_i/N_{i+1}) - \ln(\bar{N}_i/\bar{N}_{i+1})$$

$$= \ln(1 + \rho(N_i)) - \ln(1 + \rho(N_{i+1}))$$

$$\simeq \rho(N_i) - \rho(N_{i+1})$$

using (4)

$$F_i - \mathcal{F}_i = \rho(N_i) - \rho(N_i)e^{\mathcal{F}_i}$$

$$\rho(F_i) = \rho(N_i)\,\frac{1 - e^{\mathcal{F}_i}}{\mathcal{F}_i} \qquad (5)$$

Fig 1 shows $\rho(F_i)/\rho(N_i)$ plotted against $\mathcal{F}_i$

(5) Differs from eqn. 2.7 in Pope (1971).

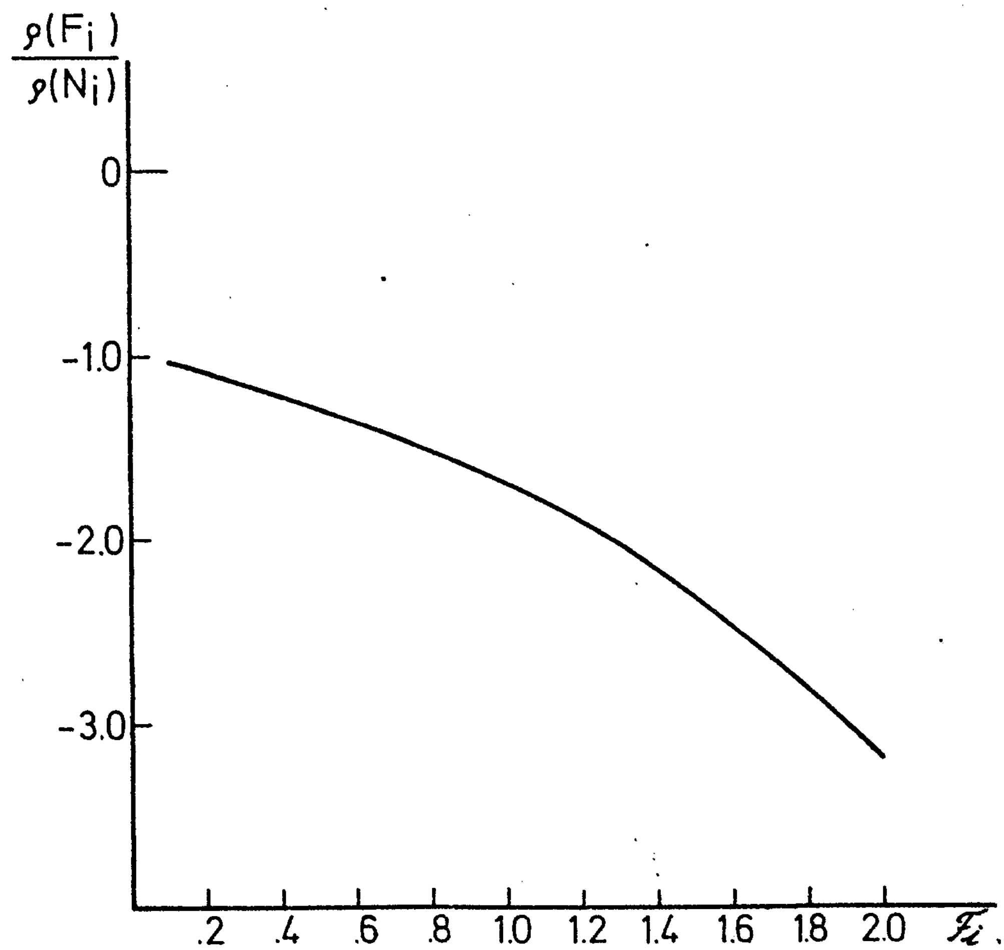

Figure 1 $\frac{\rho(F_i)}{\rho(N_i)}$ **as function of** $\mathcal{F}_i$ **(see (5))**

4 The natural mortality M

4.1. The fishing mortality F_i. Bias from M.

The influence of uncertainty in the natural mortality on the VPA was investigated by selecting $\mathcal{F}_i$ and $\mathcal{M}$ (i=0 to t) for which the corresponding catches were computed. By choosing different values of M and F_t the errors are directly calculated.

Simulation of catches with a given sampling error for a set of $\mathcal{F}_i$ (i=0 to t) and $\mathcal{M}$ is a simple way to tabulate the corresponding errors in the estimates of F_i and N_i arising from the VPA.

The following tabulations were carried out:

1) The catches were computed for $\mathcal{M}$ = .2 and $\mathcal{F}_i$ = .1,.3,.4,.5,.5, .6,.6,.7, and .7; t = 9.

F_i was estimated for five different initial guesses of E_t and with M values of .1 and .3.

The errors found, which are the biases on the estimate of F_i, are shown in fig. 2 as percentage deviation from $\mathcal{F}_i$.

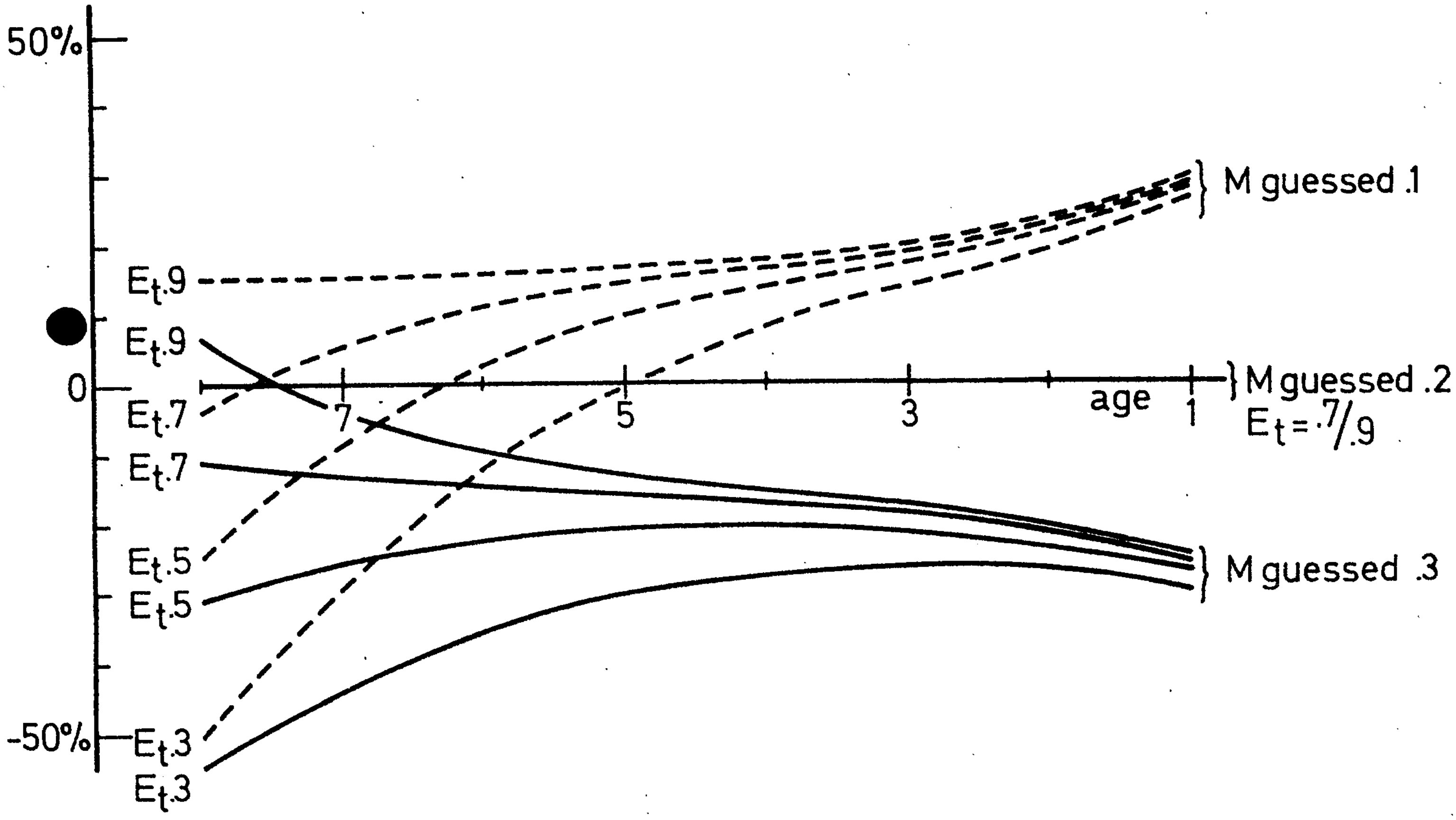

Figure 2 (F_i) in pct. five different E_t , M = .1, .3 and $\mathcal{M}$ = .2

If M is overestimated, F_i will nearly always be underestimated and converge towards too low values independent of the initial E_t values. An underestimate of M gives the opposite result.

A low fishing mortality or a short exploited phase will enlarge this effect sharply.

2) The effect of the $\mathcal{F}_i/\mathcal{M}$ ratio is demonstrated in fig. 3, where $\mathcal{F}_i$ = .7 for all i and in fig. 4 where $\mathcal{F}_i$ = .1 for all i. On both figures $\mathcal{M}$ = M = .2.

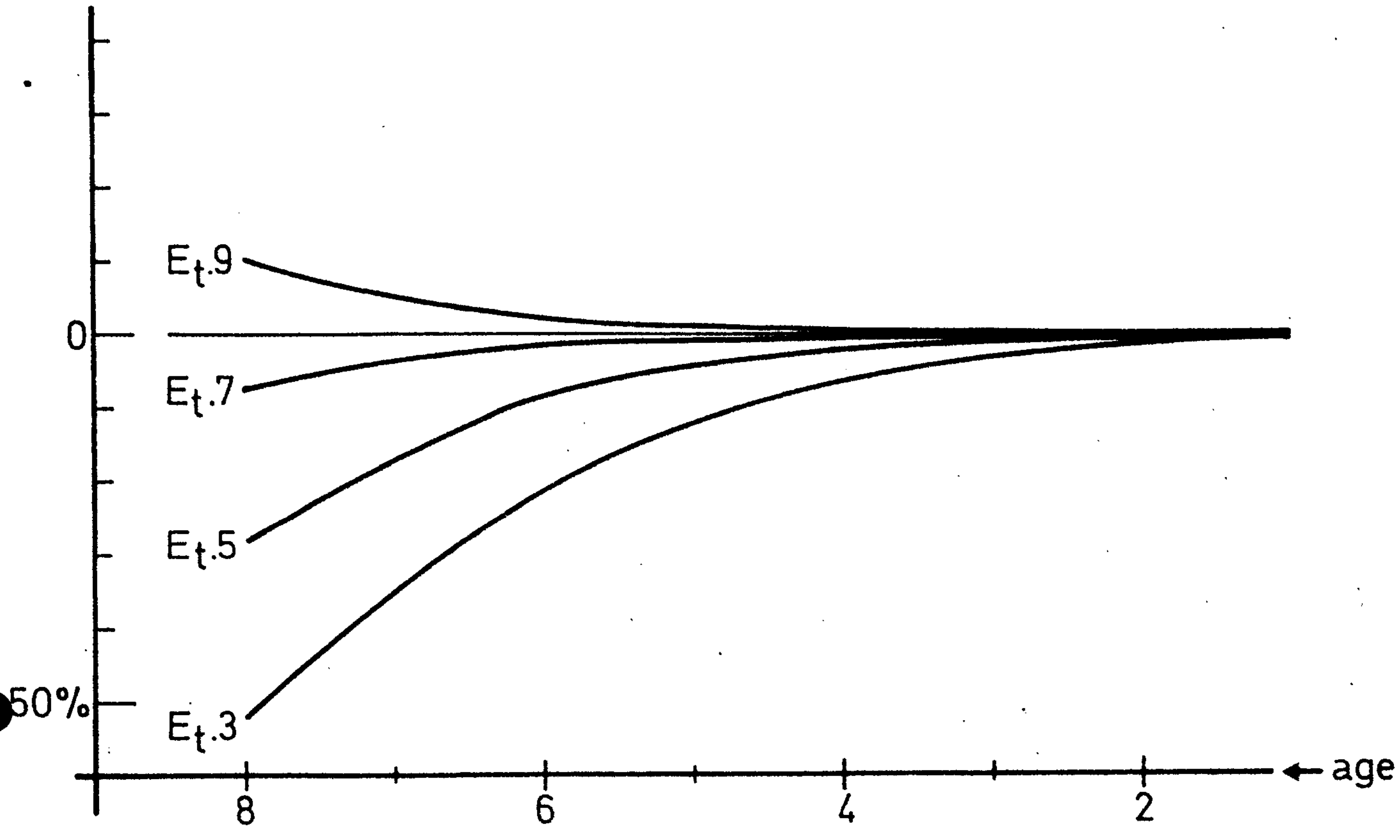

Figure 3 (F_i) in per cent for five different E_t and $\mathcal{M} = M = .2$.
$\mathcal{F}_i = .7$ for all i.

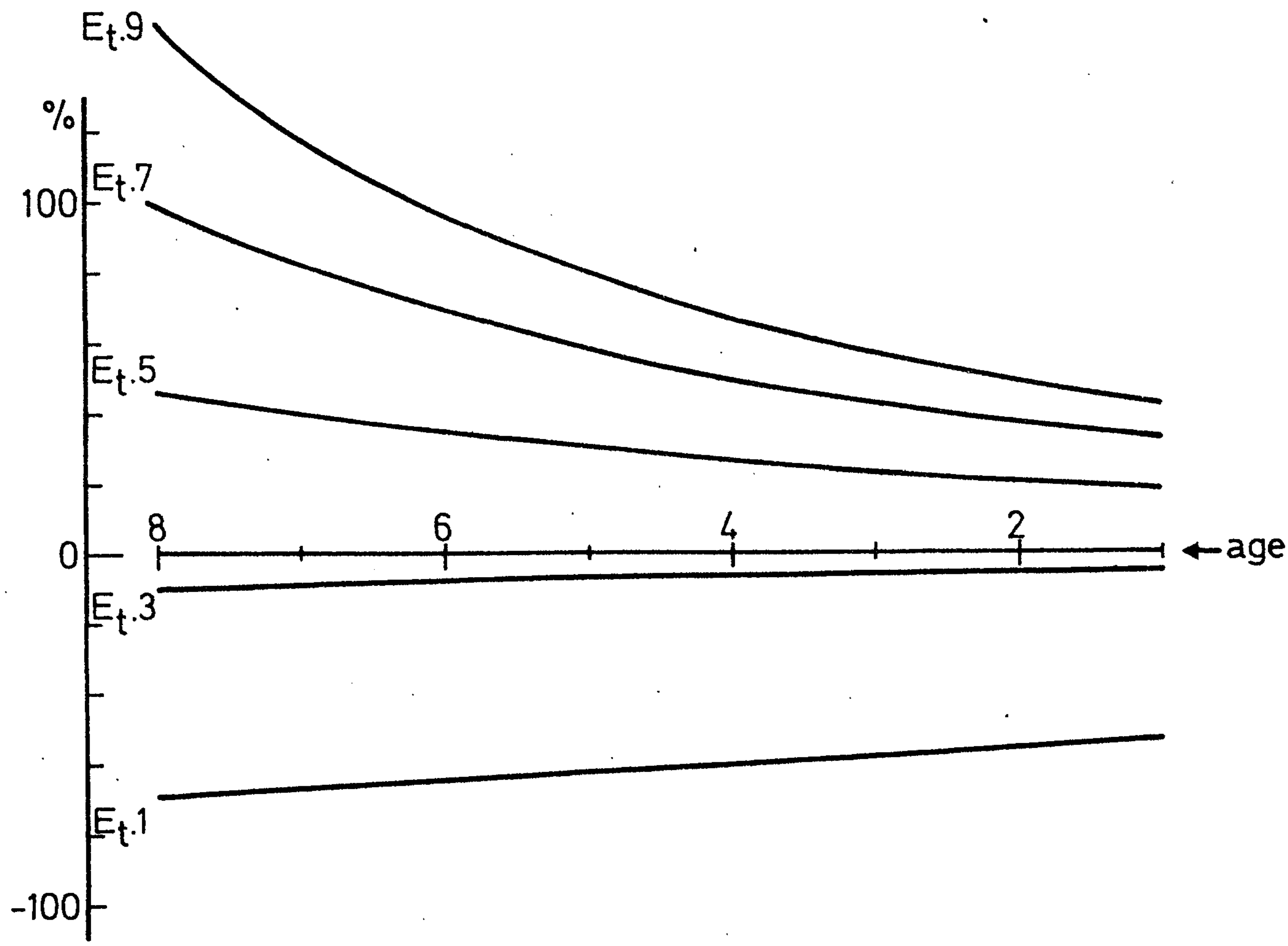

Figure 4 (F_i) in per cent for five different E_t and $\mathcal{M} = M = .2$.
$\mathcal{F}_i = .1$ for all i.

4.2 The Mean Fishing Mortality $\bar{F}^y_a$. Bias from M and E_t. Errors in $\bar{F}^y_a$ from sampling error in the catch.

$\bar{F}^y_a$ is calculated for constant recruitment, $\mathcal{F}_i$ = .1,.3,.4,.6,.6,.6,.6,.6, $\mathcal{M}$ = .2 ; a = 4 ;
The influence of an incorrect guess on M and E_t is demonstrated in figure 5.

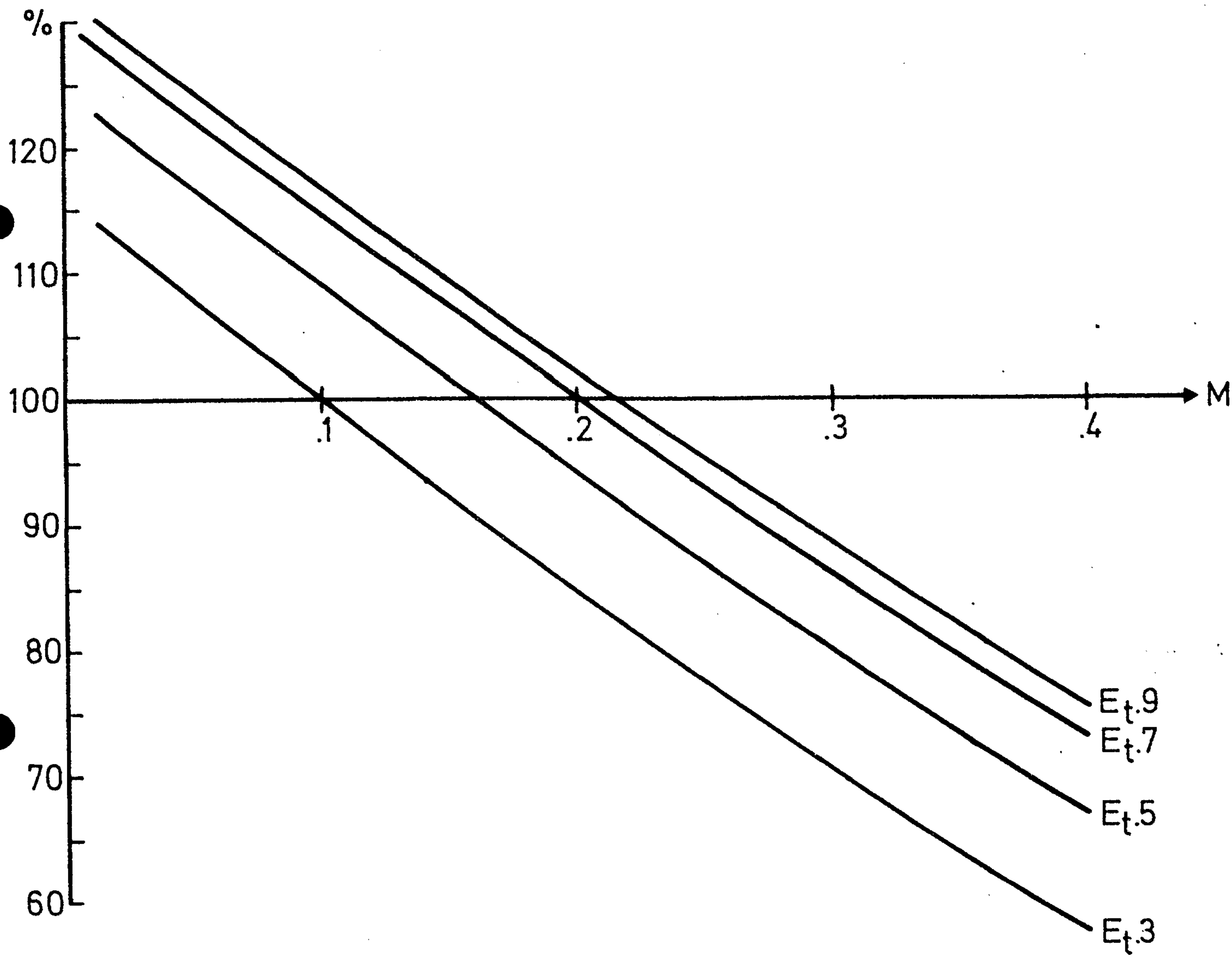

Figure 5 $\bar{F}^y_4$ as percentage of the constant fishing mortality of the old fish as function of M and of E_t. $\mathcal{M}$ = .2 $\bar{F}^y_4$ is calculated by weighting the F_i with catches.

The sampling error in the catches will introduce error in $\bar{F}_a^y$. This is investigated using a simulation technique.
The set of F_i given above was used and $\bar{F}_4^y$ was computed using means of F_i weighted by the catches. The result is summarized in the table below.

	STANDARD MEAN DEVIATION IN CATCHES 2.5 %	5 %	10 %
Standard Mean Deviation in $\bar{F}_4^y$	1.3 %	2.8 %	5.2 %

Each simulation consists of 5000 experiments and the pseudorandom number generator for the RC7000 was used together with a conversion of equally distributed numbers into normal distributed numbers (Abramovits and Stegun).

4.3 Estimation of M and $\bar{F}_a^y$ by Least Square Estimation.

In the previous section (4.1) the importance of knowing M is demonstrated. In this section an attempt to estimate M and $\bar{F}_a^y$ is given and thus give a better starting point for the VPA.

The method is a least square estimation of M and $\bar{F}_a^y$ from catch data split by yearclass and age. It will only give an estimate of M for fish elder than say a_o, chosen in such a way that the fishing mortality can be assumed constant in a given year for fish elder than a_o years.

First the least square expression is set up neglecting correlation between the catches.

$$\sum_{\substack{i \in \text{yearclass} \\ y=i+a-a_o}} \quad \sum_{\substack{a \geq a_o \\ a \in \text{age}}} \left[C_i^y \text{ (observed)} - C_i^y \text{ (calculated)} \right]^2 = \text{minimum} \quad (6)$$

C_i^y is calculated as

$$C_i^y = N_i^y E^y (1 - e^{-\bar{F}_{a_o}^y - M})$$

where

$$N_i^y = N_i^{y-1} e^{-\bar{F}_{a_o}^y - M}$$

and

$$E^y = \frac{\bar{F}_{a_o}^y}{\bar{F}_{a_o}^y + M}$$

The minimum of (6) is found by standard techniques.
The problem of correlation between the catches was neglected. It is obvious that correlation exists but not much is known about it.

5. Conclusion.

The error introduced in the VPA due to uncertainties in the natural mortality has been investigated. It is shown that a bias on the estimates of F_i and N_i of about 25 % is achieved for the normal interval of choosing M, [. 1; .3] se fig. 2.

The errors in $\bar{F}_a^y$ due to sampling error in the catches were investigated and showed that about half the relative standard deviation of the catches is found as an error in $\bar{F}_a^y$.

A method for estimation of M and $\bar{F}_a^y$ using a least square fit is introduced.

6. Litterature.

Gulland,. J.A. Estimation of Mortality Rates, Annex to Artic Fisheries Working Group Report ICES C.M. 1965.

Pope, J.G., An investigation of the Accuracy of the Virtual Population Analysis ICNAF Res. Doc. 1971.

Abramowits, M. and I.A.Stegun, Handbook of Mathematical Functions, 1965. Dover Pub. Inc. New York.